2226 – 5025M

Markscheme

May 2026

Mathematics: analysis and approaches

Higher level

Paper 1

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite student errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some students may be advantaged for that specific question item, it is likely that these students will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

  Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*

2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that students may be using an incorrect value.

- If the student’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when students fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a student makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a student incorrectly copies values or information from the question, this is a mis-read (***MR***). A student should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of students' own work does **not** constitute a misread, it is an error.
- If a student uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Students will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the student work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however students may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Students are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a student has used any calculator, please follow the procedures for malpractice.

10 Presentation of student work

Crossed out work: If a student has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the student indicates that they would like the work to be marked.

More than one solution: Where a student offers two or more different answers to the same question, an examiner should only mark the first response unless the student indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) attempt to find the correct proportion of Grade 12 students in the sample ***(M1)***

$\frac{170}{600}\times 60$ OR $\frac{60}{600}\times 170$ OR $\frac{170}{600}=\frac{x}{60}$ OR $\left(\frac{170}{600}=\right)\frac{17}{60}$ (or equivalent)

(number of Grade 12 =) 17 ***A1***

[2 marks]

(b) 80 ***A1***

[1 mark]

(c) $a=90$ ***A1***

[1 mark]

(d) attempt to use outlier formula ***(M1)***

$66-1.5\times 24$

smallest amount is $\$30$ ***A1***

Note: If they find 30, then state that 30 is an outlier and/or gives a final answer other than 30 (usually 31), award ***M1A0***.

[2 marks]

Total [6 marks]

2. (a) attempt to use cosine rule ***(M1)***

$\mathrm{AC}^2=5^2+6^2-2\times5\times6\times\frac{1}{5}\left(=25+36-2\times5\times6\times\frac{1}{5}=49\right)$ ***A1***

$\mathrm{AC}=7\text{ cm}$ ***A1***

[3 marks]

(b) (i) attempt to use Pythagoras ***(M1)***

$\sin^2 \mathrm{A\hat{B}C}+\left(\frac{1}{5}\right)^2=1$ **OR** $x^2+1^2=5^2$ **OR** right triangle with side 1 and hypotenuse 5

$\sin^2 \mathrm{A\hat{B}C}=\frac{24}{25}$ **OR** $\sin \mathrm{A\hat{B}C}=\sqrt{\frac{24}{25}}\left(=\frac{\sqrt{24}}{\sqrt{25}}=\frac{\sqrt{24}}{5}=\frac{\sqrt{4}\sqrt{6}}{5}\right)$ ***A1***

$=\frac{2\sqrt{6}}{5}$ ***AG***

(ii) area of triangle $\mathrm{ABC}=\frac{1}{2}\times5\times6\times\frac{2\sqrt{6}}{5}$ ***(A1)***

$=6\sqrt{6}\ \left(=3\sqrt{24}\right)\text{ cm}^2$ ***A1***

[4 marks]

Total [7 marks]

3. (a) $\log_3 p = 3 + \log_3 r$

use of at least one "log rule" or exponential form applied correctly ***(A1)***

$\log_3\left(\frac{p}{r}\right) = 3$ **OR** $p = 3^{3+\log_3 r}$ **OR** $\log_3 p = \log_3 3^3 + \log_3 r$ **OR** $\log_3 p = 3\log_3 3 + \log_3 r$

$\log_3 p = \log_3 27r$ **OR** $\frac{p}{r} = 3^3$ **OR** $p = 3^3 r$ **OR** $\frac{p}{r} = 27$ ***A1***

$p = 27r$ ***AG***

[2 marks]

(b) **METHOD 1**

substitution of $p = 27r$ into the second equation ***(M1)***

$\log_r 27r = 4$

use of at least one "log rule" or exponential form applied correctly for the second equation ***(A1)***

$27r = r^4$ **OR** $\log_r 27 + 1 = 4 \left(\Rightarrow \log_r 27 = 3\right)$

$r^3 = 27$

$r = 3$ ***A1***

$p = 81$ ***A1***

METHOD 2

use of at least one "log rule" or exponential form applied correctly for the second equation ***(A1)***

$\log_r p = 4 \Rightarrow p = r^4$

substitution of $p = 27r$ into $p = r^4$ ***(M1)***

$27r = r^4$

$r^3 = 27$

$r = 3$ ***A1***

$p = 81$ ***A1***

[4 marks]

Total [6 marks]

4. **METHOD 1**

EITHER

attempt to use compound angle identities with two appropriate values ***(M1)***

$\cos(A+B)=\cos A\cos B-\sin A\sin B$, where $A+B=75^\circ$

$\cos(45^\circ+30^\circ)=\cos 45^\circ\cos 30^\circ-\sin 45^\circ\sin 30^\circ$ ***(A1)***

$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\ \left(=\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ **OR** $=\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\times\frac{1}{2}\ \left(=\frac{\sqrt{6}-\sqrt{2}}{4}\right)$ ***A1***

OR

attempt to use compound angle identities with two appropriate values ***(M1)***

$\cos(A-B)=\cos A\cos B+\sin A\sin B$, where $A-B=75^\circ$

$\cos(120^\circ-45^\circ)=\cos 120^\circ\cos 45^\circ+\sin 120^\circ\sin 45^\circ$ ***(A1)***

$=-\frac{1}{2}\times\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}\ \left(=\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ **OR** $=-\frac{1}{2}\times\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}\ \left(=\frac{\sqrt{6}-\sqrt{2}}{4}\right)$ ***A1***

THEN

attempt to take reciprocal and rationalise their denominator ***(M1)***

$\sec 75^\circ=\frac{1}{\cos 75^\circ}=\frac{2\sqrt{2}}{\left(\sqrt{3}-1\right)}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$ **OR** $\sec 75^\circ=\frac{1}{\cos 75^\circ}=\frac{4}{\left(\sqrt{6}-\sqrt{2}\right)}\times\frac{\left(\sqrt{6}+\sqrt{2}\right)}{\left(\sqrt{6}+\sqrt{2}\right)}$

$=\sqrt{6}+\sqrt{2}$ ***A1***

continued…

Question 4 continued

METHOD 2

EITHER

attempt to use double angle identities with an appropriate value of θ ***(M1)***

$\cos 75^\circ = \sqrt{\dfrac{1+\cos 150^\circ}{2}}$ **OR** $\sin 15^\circ = \sqrt{\dfrac{1-\cos 30^\circ}{2}}$

$\cos 75^\circ = \sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}} \left(=\dfrac{1}{2}\sqrt{2-\sqrt{3}}\right)$ ***A1***

attempt to rationalise their denominator ***(M1)***

$\sec 75^\circ = \dfrac{2}{\sqrt{2-\sqrt{3}}} \times \dfrac{\sqrt{2+\sqrt{3}}}{\sqrt{2+\sqrt{3}}}$

OR

attempt to use double angle identities with an appropriate value of θ ***(M1)***

$\cos 15^\circ = \sqrt{\dfrac{1+\cos 30^\circ}{2}}$

attempt to use double angle identity $\sin 2\theta = 2\sin\theta\cos\theta$ with appropriate value of θ ***(M1)***

$\sin 30^\circ = 2\sin 15^\circ \sqrt{\dfrac{1+\frac{\sqrt{3}}{2}}{2}} \left(\Rightarrow \sin 15^\circ = \dfrac{1}{2\sqrt{2+\sqrt{3}}}\right)$

$\cos 75^\circ = \dfrac{1}{2\sqrt{2+\sqrt{3}}}$ ***A1***

THEN

$\sec 75^\circ = 2\sqrt{2+\sqrt{3}} \left(=\sqrt{8+4\sqrt{3}}\right)$ ***A1***

$= \sqrt{\left(\sqrt{6}+\sqrt{2}\right)^2}$

$= \sqrt{6}+\sqrt{2}$ ***A1***

[5 marks]

5. (a) $\frac{dy}{dx} = 4x^3 - 4\cos 4x$ ***A1A1***

Note: Award ***A1*** for each correct term. Award at most ***A1A0*** if additional terms are seen.

[2 marks]

(b) **EITHER**

integrating by inspection ***(M1)***

$\frac{1}{4}\int (x^4 - \sin 4x)^7 (4x^3 - 4\cos 4x)\ dx$ OR $\frac{\left(x^4 - \sin 4x\right)^8}{8}$ ***(A1)***

OR

integrating by substitution ***(M1)***

$u = x^4 - \sin 4x \left(\Rightarrow \frac{du}{dx} = 4x^3 - 4\cos 4x\right)$

$\frac{1}{4}\int u^7 du$ OR $\frac{u^8}{8}$ ***(A1)***

$\frac{1}{32}u^8\left(+c\right)$

THEN

correct integrated expression in terms of x

$\frac{1}{32}\left(x^4 - \sin 4x\right)^8\ \left(+c\right)$ ***A1***

[3 marks]

Total [5 marks]

6. (a) **EITHER**

attempt to interchange x and y (seen anywhere) ***(M1)***

$3y+2=\dfrac{-7}{x-4}$ OR $3xy-12y=-2x+1$ (or equivalent, with y-term(s) on one side) ***(A1)***

OR

$3x+2=\dfrac{-7}{y-4}$ OR $3xy-12x=-2y+1$ (or equivalent, with x-term(s) on one side) ***(A1)***

attempt to interchange x and y (seen anywhere) ***(M1)***

THEN

$f^{-1}(x)=\dfrac{-7}{3(x-4)}-\dfrac{2}{3}$ **OR** $f^{-1}(x)=\dfrac{1-2x}{3x-12}$ $(x\neq 4)$ (or equivalent) (accept $y=$) ***A1***

[3 marks]

(b) **METHOD 1**

attempt to use $f^{-1}(x)$ on both sides ***(M1)***

$g\left(\dfrac{x}{3}\right)=f^{-1}(x)$

$g(x)=f^{-1}(3x)$ ***(A1)***

$g(x)=\dfrac{-7}{3(3x-4)}-\dfrac{2}{3}$ **OR** $g(x)=\dfrac{1-6x}{9x-12}$ $\left(x\neq\dfrac{4}{3}\right)$ (or equivalent) (accept $y=$) ***A1***

METHOD 2

attempt to substitute

eg. $a=\dfrac{x}{3}$ **OR** $f(g(a))=3a$ **OR** $g(a)=f^{-1}(3a)$ ***(M1)***

$g(a)=\dfrac{1-6a}{9a-12}$ ***(A1)***

$g(x)=\dfrac{1-6x}{9x-12}$ $\left(x\neq\dfrac{4}{3}\right)$ (or equivalent) (accept $y=$) ***A1***

[3 marks]

Total [6 marks]

7. (a) $S_k = \frac{1}{2} S_\infty$

$\frac{u_1\left(1-r^k\right)}{1-r} = \frac{1}{2}\left(\frac{u_1}{1-r}\right)$ (or equivalent, seen anywhere) ***A1***

EITHER

attempt to form an equation in r^k by eliminating u_1 and $1-r$ ***(M1)***

$1-r^k = \frac{1}{2}$

$r^k = \frac{1}{2}$ ***A1***

$\frac{1}{16} = \frac{1}{2}u_1$ ***A1***

OR

$u_1 r^k = \frac{1}{16}$ ***A1***

attempt to form an equation in u_1 by eliminating $1-r$ ***(M1)***

$u_1\left(1-\frac{1}{16u_1}\right) = \frac{1}{2}u_1$

$\frac{1}{16u_1} = \frac{1}{2}$ ***A1***

THEN

$u_1 = \frac{1}{8}$ ***AG***

Note:	Do not award any marks for students starting with $u_1 = \frac{1}{8}$ to verify the result is true.

[4 marks]

(b) $u_{2k+1} = u_1\left(r^k\right)^2$ **OR** $u_{2k+1} = u_1\left(r^{2k}\right)$ **OR** $u_{2k+1} = \frac{1}{8}\times\left(\frac{1}{2}\right)^2$ ***(A1)***

$u_{2k+1} = \frac{1}{32}$ ***A1***

[2 marks]
Total [6 marks]

8. $\arg(z) = \arctan\left(\frac{k}{3}\right)$ **OR** $k = 3\tan\theta$ (seen anywhere) ***(A1)***

$\mathrm{Im}\left(z^5\right) = 0$ **OR** $\arg\left(z^5\right) = 0, (-\pi, -2\pi, \ldots)$ ***(A1)***

use of De Moivre's theorem to find $\arg\left(z^5\right)$ ***(M1)***

$\arg\left(z^5\right) = 5 \times \arg(z)$

z lies in the 4th quadrant and $\frac{-\pi}{2} < \arg z < 0 \Rightarrow \frac{-5\pi}{2} < 5\arg z < 0$

$5 \times \arg(z) = -\pi, -2\pi$

$\arg(z) = (\pm)\frac{\pi}{5}$ **OR** $(\pm)\frac{2\pi}{5}$ (accept degree equivalents) ***(A1)***

EITHER

$\arg(z) = -\frac{\pi}{5}$ **AND** $-\frac{2\pi}{5}$ ***A1***

OR

$k = 3\tan\left(-\frac{\pi}{5}\right)$ **OR** $3\tan\left(-\frac{2\pi}{5}\right)$ ***A1***

THEN

$k = 3\tan\left(-\frac{\pi}{5}\right), 3\tan\left(-\frac{2\pi}{5}\right)$ (for both correct and no extra solutions) ***A1***

[6 marks]

9. **METHOD 1**

base case, $n=2, \frac{1}{2+1}+\frac{1}{2+2}=\frac{7}{12}\left(\geq \frac{7}{12}\right)$, so true for $n=2$ ***R1***

Note: Subsequent marks after this ***R1*** are independent of this mark and can be awarded.

assume true for $n=k$, i.e. $\frac{1}{k+1}+\frac{1}{k+2}+\frac{1}{k+3}+\cdots+\frac{1}{2k} \geq \frac{7}{12}$ ***M1***

Note: Award ***M0*** for statements such as "let $n=k$" or "$n=k$ is true". The assumption of truth must be clear. Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

considers $n=k+1$

$\frac{1}{k+2}+\frac{1}{k+3}+\frac{1}{k+4}+\cdots+\frac{1}{2k+2}$ ***(M1)***

$\geq \frac{7}{12}-\frac{1}{k+1}+\frac{1}{2k+1}+\frac{1}{2k+2}$ (or equivalent) ***A1***

attempt to combine the expressions in k ***(M1)***

$-\frac{2}{2k+2}+\frac{1}{2k+1}+\frac{1}{2k+2}$

EITHER

$\frac{7}{12}+\frac{1}{(2k+1)(2k+2)}$ (or equivalent) ***A1***

$\geq \frac{7}{12}$ (since $k>0$) ***A1***

OR

$\frac{7}{12}+\frac{1}{2k+1}-\frac{1}{2k+2}$ (or equivalent) ***A1***

$\geq \frac{7}{12}$ since $\frac{1}{2k+1}>\frac{1}{2k+2}$ (and $k>0$) ***A1***

THEN

since true for $n=2$, and true for $n=k$ implies true for $n=k+1$,
therefore true for all $n \in \mathbb{Z}^{+}, n \geq 2$ ***R1***

Note: Only award ***R1*** if 4 of the previous 7 marks have been awarded.

continued…

Question 9 continued

METHOD 2

base case, $n = 2, \frac{1}{2+1} + \frac{1}{2+2} = \frac{7}{12}\left(\geq \frac{7}{12}\right)$, so true for $n = 2$ ***R1***

Note:	Subsequent marks after this ***R1*** are independent of this mark and can be awarded.

assume true for $n = k$, i.e. $\frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \cdots + \frac{1}{2k} \geq \frac{7}{12}$ ***M1***

Note:	Award ***M0*** for statements such as "let $n = k$" or "$n = k$ is true". The assumption of truth must be clear. Subsequent marks after this ***M1*** are independent of this mark and can be awarded.

considers $n = k + 1$

$\frac{1}{k+2} + \frac{1}{k+3} + \frac{1}{k+4} + \cdots + \frac{1}{2(k+1)}$ ***(M1)***

$\left(2k+1 < 2k+2 \Rightarrow\right) \frac{1}{2k+1} > \frac{1}{2}\left(\frac{1}{k+1}\right)$ (or equivalent) ***A1***

$\frac{1}{2k+1} + \frac{1}{2}\left(\frac{1}{k+1}\right) > \frac{1}{k+1}$ (or equivalent) ***(M1)***

$\frac{1}{k+2} + \frac{1}{k+3} + \frac{1}{k+4} + \cdots + \frac{1}{2k} + \frac{1}{2k+1} + \frac{1}{2k+2}$

$> \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \frac{1}{k+4} + \cdots + \frac{1}{2k}$ (or equivalent) ***A1***

$\geq \frac{7}{12}$ (since $k > 0$) ***A1***

since true for $n = 2$, and true for $n = k$ implies true for $n = k + 1$,
therefore true for all $n \in \mathbb{Z}^+, n \geq 2$ ***R1***

Note: Only award ***R1*** if 4 of the previous 7 marks have been awarded.

[8 marks]

Section B

10. (a) **METHOD 1**

attempt to use a double angle formula for $\sin 2x$ **OR** $\cos 2x$ *(M1)*

$2\sin x\cos x = 1-\left(1-2\sin^2 x\right)$ **OR** $2\sin x\cos x = 1-\left(2\cos^2 x-1\right)$

OR $2\sin x\cos x = 1-\left(\cos^2 x-\sin^2 x\right)$

$2\sin x\cos x = 2\sin^2 x$ *(A1)*

attempt to factorize *(M1)*

$2\sin x\left(\cos x-\sin x\right)=0$

recognition of $\sin x = 0$ **OR** $\sin x = \cos x\left(\Rightarrow \tan x = 1\right)$ *(M1)*

$x = 0, \dfrac{\pi}{4}, \pi$ ***A2***

METHOD 2

attempt to square both sides of the equation *(M1)*

$\sin^2 2x = \left(1-\cos 2x\right)^2$

$\sin^2 2x = 1-2\cos 2x+\cos^2 2x$ *(A1)*

substitute $\sin^2 2x = 1-\cos^2 2x$ in their equation *(M1)*

$1-\cos^2 2x = 1-2\cos 2x+\cos^2 2x \quad \left(\Rightarrow \cos^2 2x-\cos 2x = 0\right)$

recognition of $\cos 2x = 0$ **OR** $\cos 2x = 1$ *(M1)*

$x = 0, \dfrac{\pi}{4}, \pi$ ***A2***

METHOD 3

attempt to square both sides of the equation $\sin 2x+\cos 2x = 1$ *(M1)*

$\left(\sin 2x+\cos 2x\right)^2 = 1$

$\sin^2 2x+\cos^2 2x+2\sin 2x\cos 2x = 1$ *(A1)*

substitute $\sin^2 2x+\cos^2 2x = 1$ OR $2\sin 2x\cos 2x = \sin 4x$ into their equation *(M1)*

$1+\sin 4x = 1$

$\sin 4x = 0$ *(A1)*

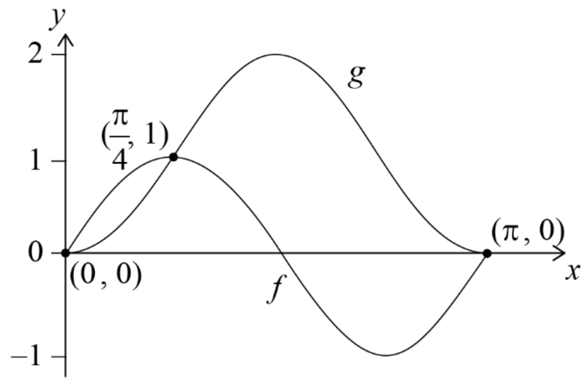
$x = 0, \dfrac{\pi}{4}, \pi$ ***A2***

[6 marks]

continued…

Question 10 continued

(b)



correct graph of $y=\sin 2x$ ***A1***

attempt to sketch a negative cosine graph or cosine graph of period π ***(M1)***

correct graph of $y=1-\cos 2x$ ***A1***

both correct graphs sketched as curves on the same axes within the given domain and identifying points of intersection at $(0,0),\left(\frac{\pi}{4},1\right)$ and $(\pi,0)$ ***A1***

[4 marks]

(c) recognizing to integrate $(\pm)(1-\cos 2x-\sin 2x)$ ***(M1)***

$\int_{\frac{\pi}{4}}^{\pi}(1-\cos 2x-\sin 2x)\,\mathrm{d}x$ **OR** $\int_{\frac{\pi}{4}}^{\pi}(1-\cos 2x-2\sin x\cos x)\,\mathrm{d}x$

$=\left[x-\frac{\sin 2x}{2}+\frac{\cos 2x}{2}\right]_{\frac{\pi}{4}}^{\pi}$ **OR** $=\left[x-\frac{\sin 2x}{2}-\sin^2 x\right]_{\frac{\pi}{4}}^{\pi}$ (or equivalent) ***A1A1***

Note: Award ***A1*** for $x-\frac{\sin 2x}{2}$ and ***A1*** for $\frac{\cos 2x}{2}$ **OR** $-\sin^2 x$.

attempt to substitute their limits into their integral and subtract ***(M1)***

$=\left(\pi-\frac{1}{2}\sin 2\pi+\frac{1}{2}\cos 2\pi\right)-\left(\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{2}+\frac{1}{2}\cos\frac{\pi}{2}\right)$ **OR**

$\left(\pi-\frac{1}{2}\sin 2\pi-\sin^2\pi\right)-\left(\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{2}-\sin^2\frac{\pi}{4}\right)$

$=\left(\pi-0+\frac{1}{2}\right)-\left(\frac{\pi}{4}-\frac{1}{2}+0\right)$ **OR** $=(\pi-0-0)-\left(\frac{\pi}{4}-\frac{1}{2}-\frac{1}{2}\right)$ (or equivalent) ***A1***

area $=\frac{3}{4}\pi+1$ ***A1***

[6 marks]
Total [16 marks]

11. (a) **METHOD 1**

(i) recognize L_1 is perpendicular to the normal of Π ***(M1)***

$\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ a \end{pmatrix} = 0$

$2 + a = 0$ ***A1***

$a = -2$ ***AG***

(ii) Substitute $(7, -1, 1)$ and $a = -2$ into Π ***(M1)***

$2(7) + (-1) + (-2) \neq b$ leading to

$b \neq 11$ ***A1***

METHOD 2

(i) attempt to find the parametric equations of L_1 ***(M1)***

$x = 7, y = -1 + 2\lambda, z = 1 + \lambda$

attempt to substitute the parametric equations into Π ***(M1)***

$2(7) + (-1 + 2\lambda) + a(1 + \lambda) = b$

$13 + a + (2 + a)\lambda = b$ **OR** $\lambda = \dfrac{b - 13 - a}{2 + a}$ ***A1***

non-unique solution when $2 + a = 0$

$a = -2$ ***AG***

(ii) infinite solutions when $a + 13 = b$

(as L_1 does not lie on Π) $b \neq 11$ ***A1***

[4 marks]

(b) $L_2 : \mathbf{r} = \begin{pmatrix} 7 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ **OR** $x = 7 + \mu, y = -1 + \mu, z = 1 - \mu$ (seen anywhere) ***(A1)***

attempt to substitute L_2 into Π ***(M1)***

$2(7 + \mu) + (-1 + \mu) - 2(1 - \mu) = 2$

$\mu = -\dfrac{9}{5}$ ***A1***

attempt to substitute their μ into L_2 ***(M1)***

coordinates are $\left(\dfrac{26}{5}, \dfrac{-14}{5}, \dfrac{14}{5} \right)$ ***A1***

[5 marks]

continued…

Question 11 continued

(c) **METHOD 1**

attempt to use angle between two vectors formula ***(M1)***

$(\cos\theta =)\dfrac{5}{3\sqrt{3}}$ ***(A1)***

recognition that $\alpha = 90^\circ - \theta$ (seen anywhere) ***(M1)***

$\cos\left(90^\circ - \alpha\right) = \dfrac{5}{3\sqrt{3}}$

$\sin\alpha = \dfrac{5}{3\sqrt{3}}$ ***A1***

METHOD 2

attempt to find a cross product ***(M1)***

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$$

$(\sin\theta =)\dfrac{\sqrt{2}}{3\sqrt{3}}$ ***(A1)***

recognition that $\alpha = 90^\circ - \theta$ (seen anywhere) ***(M1)***

$\sin\left(90^\circ - \alpha\right) = \dfrac{\sqrt{2}}{3\sqrt{3}}$ $\left(\cos\alpha = \dfrac{\sqrt{2}}{3\sqrt{3}}\right)$

$\sin\alpha = \dfrac{5}{3\sqrt{3}}$ ***A1***

Note:	Accept other equivalent forms.

[4 marks]

continued…

Question 11 continued

(d) attempt to use right angled trigonometry $\sin\alpha = \dfrac{d}{\text{AN}}$ (or equivalent) ***(M1)***

$d = \text{AN} \times \sin\alpha = \dfrac{9\sqrt{3}}{5} \times \dfrac{5}{3\sqrt{3}}$

$= 3$ ***A1***

[2 marks]

(e) attempt to find the parametric equations of line through A normal to Π ***(M1)***

$x = 7 + 2t, y = -1 + t, z = 1 - 2t$

attempt to substitute the parametric equations into Π ***(M1)***

$2(7 + 2t) + (-1 + t) - 2(1 - 2t) = 2$

so $t = -1$ ***(A1)***

for reflection in plane $t = -2$ OR finding midpoint ***(M1)***

$A'(3, -3, 5)$ OR $\dfrac{7 + x}{2} = 5, \dfrac{-1 + y}{2} = -2, \dfrac{1 + z}{2} = 3$

$\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ ***A1***

[5 marks]
Total [20 marks]

12. (a) **METHOD 1**

attempt to use the chain rule ***M1***

$\frac{dy}{dx} = \sec^2\left(1 - e^{3x}\right) \times \left(-3e^{3x}\right)$ ***A1A1***

Note: Award ***A1*** for $\sec^2\left(1 - e^{3x}\right)$ and ***A1*** for $-3e^{3x}$.

$\frac{dy}{dx} = -3e^{3x}(1 + \tan^2(1 - e^{3x}))$ ***A1***

$\frac{dy}{dx} = -3e^{3x}\left(1 + y^2\right)$ ***AG***

METHOD 2

attempt to take $\arctan$ on both sides ***M1***

$\arctan y = 1 - e^{3x}$

attempt to use implicit differentiation ***M1***

$\frac{1}{1+y^2}\frac{dy}{dx} = -3e^{3x}$ ***A1A1***

Note: Award ***A1*** for $\frac{1}{1+y^2}\frac{dy}{dx}$ and ***A1*** for $-3e^{3x}$.

$\frac{dy}{dx} = -3e^{3x}\left(1 + y^2\right)$ ***AG***

[4 marks]

(b) use of product rule to differentiate $-3e^{3x}\left(1 + y^2\right)$ ***(M1)***

use of implicit differentiation for $\left(1 + y^2\right)$ ***(M1)***

all terms correct ***A1***

$\frac{d^2y}{dx^2} = -9e^{3x}\left(1 + y^2\right) - 3e^{3x}\left(2y\frac{dy}{dx}\right)$

substitute $x = 0$ to find y or $\frac{dy}{dx}$

$y = 0$ **OR** $\frac{dy}{dx} = -3$ (seen anywhere) ***(A1)***

correct substitution into $\frac{d^2y}{dx^2}$ ***A1***

$\frac{d^2y}{dx^2} = -9\left(1 + 0^2\right) - 3(0)$

$\frac{d^2y}{dx^2} = -9$ ***AG***

[5 marks]

continued…

Question 12 continued

(c) attempt to use Maclaurin series formula ***(M1)***

$f(x)=0+x(-3)+\frac{x^2}{2!}(-9)$

$f(x)=-3x-\frac{9}{2}x^2$ ***A1***

[2 marks]

(d) **METHOD 1**

use of $\tan\left(1-e^{3x}\right)=-3x-\frac{9}{2}x^2-\cdots$ ***(M1)***

$\lim\limits_{x\to0}\frac{\tan\left(1-e^{3x}\right)}{x}=\lim\limits_{x\to0}\frac{-3x-\frac{9}{2}x^2}{x}$ or $\lim\limits_{x\to0}\left(-3-\frac{9}{2}x\right)$ ***(A1)***

$=-3$ ***A1***

METHOD 2

use of L'Hopital's rule ***(M1)***

$\lim\limits_{x\to0}\frac{\tan\left(1-e^{3x}\right)}{x}=\lim\limits_{x\to0}\frac{\sec^2\left(1-e^{3x}\right)\times\left(-3e^{3x}\right)}{1}\left(=\lim\limits_{x\to0}\frac{\left(1+y^2\right)\times\left(-3e^{3x}\right)}{1}\right)$ ***A1***

Note: This ***A1*** line may be seen in part (b).

$=-3$ ***A1***

METHOD 3 (may be seen after part (e))

recognize $\lim\limits_{x\to0}\frac{\tan\left(1-e^{3x}\right)}{x}=\lim\limits_{x\to0}\frac{1}{a+bx}$ ***(M1)(A1)***

$=-3$ ***A1***

[3 marks]

(e) **METHOD 1**

use of Maclaurin series or extension of binomial theorem, $n\in\mathbb{Q}$ to expand $(a+bx)^{-1}$ ***(M1)***

$g'(x)=\frac{a}{(a+bx)^2}\left(\Rightarrow g'(0)=\frac{1}{a}\right)$ OR $\frac{x}{a}\left(1+(-1)\left(\frac{b}{a}x\right)+\cdots\right)$

$=\frac{x}{a}-\frac{b}{a^2}x^2$ ***A1***

Note: Award ***M1A0*** for one correct term.

comparing coefficients of x or x^2 ***(M1)***

$a=-\frac{1}{3}$ ***A1***

$b=\frac{1}{2}$ ***A1***

continued…

Question 12 continued

METHOD 2
recognition of infinite geometric series OR attempt to find common ratio ***(M1)***

$$r = \frac{-\frac{9}{2}x^2}{-3x}$$

common ratio $r = \frac{3}{2}x$ ***A1***

substituting $u_1 = -3x$ and $r = \frac{3}{2}x$ into S_∞ formula (and equating to $\frac{x}{a+bx}$) ***(M1)***

$$S_\infty = \frac{-3x}{1-\frac{3}{2}x} \quad \left(= \frac{x}{-\frac{1}{3}+\frac{1}{2}x} \right)$$

$a = -\frac{1}{3}$ ***A1***

$b = \frac{1}{2}$ ***A1***

[5 marks]
Total [19 marks]